Cambridge O Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

ADDITIONAL MATHEMATICS **4037/13**

Paper 1 **October/November 2020**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

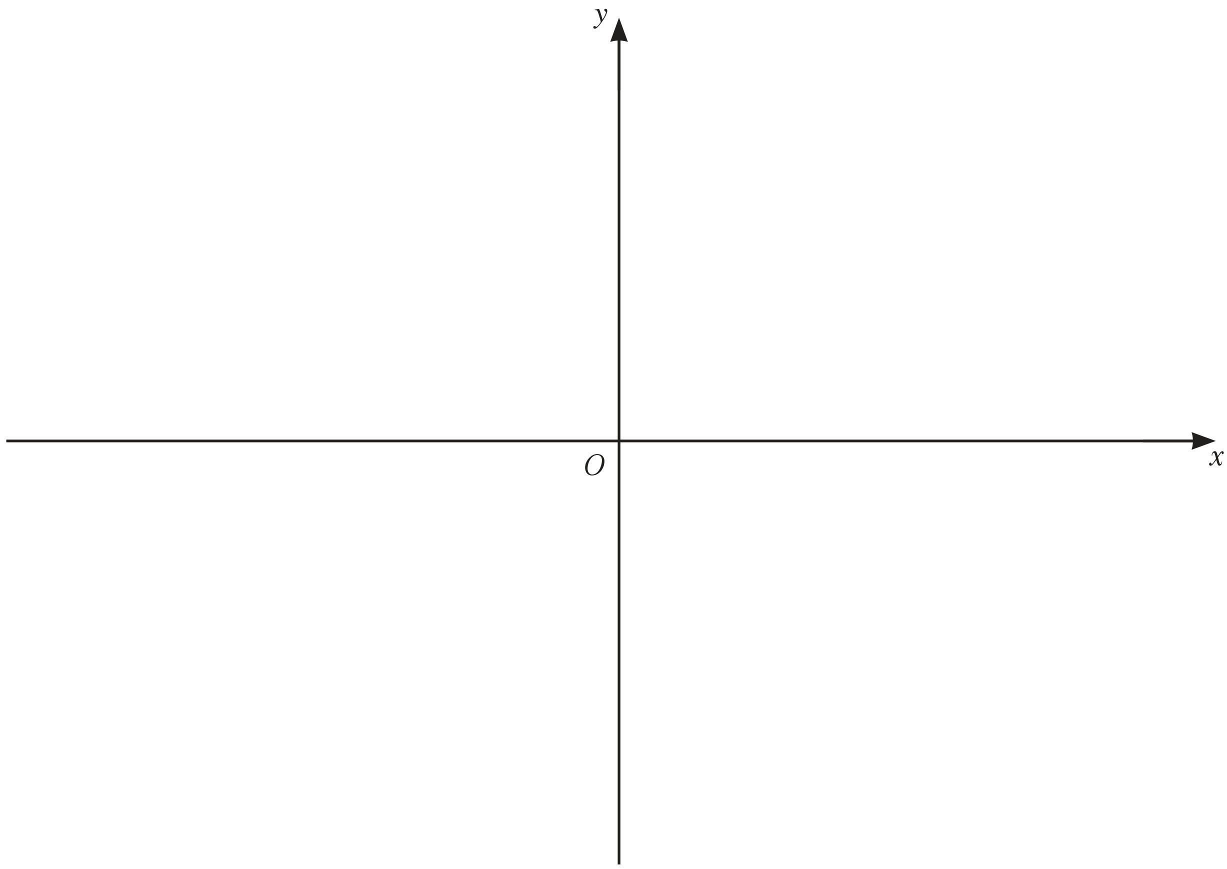
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 **(a)** On the axes below, sketch the graph of $y=(x-2)(x+1)(3-x)$, stating the intercepts on the coordinate axes.



[3]

(b) Hence write down the values of x such that $(x-2)(x+1)(3-x)>0$. [2]

2 **(a)** Given that $y = \dfrac{e^{2x-3}}{x^2+1}$, find $\dfrac{dy}{dx}$. [3]

(b) Hence, given that y is increasing at the rate of 2 units per second, find the exact rate of change of x when $x = 2$. [3]

3 **(a)** $f(x) = 4\ln(2x - 1)$

(i) Write down the largest possible domain for the function f. [1]

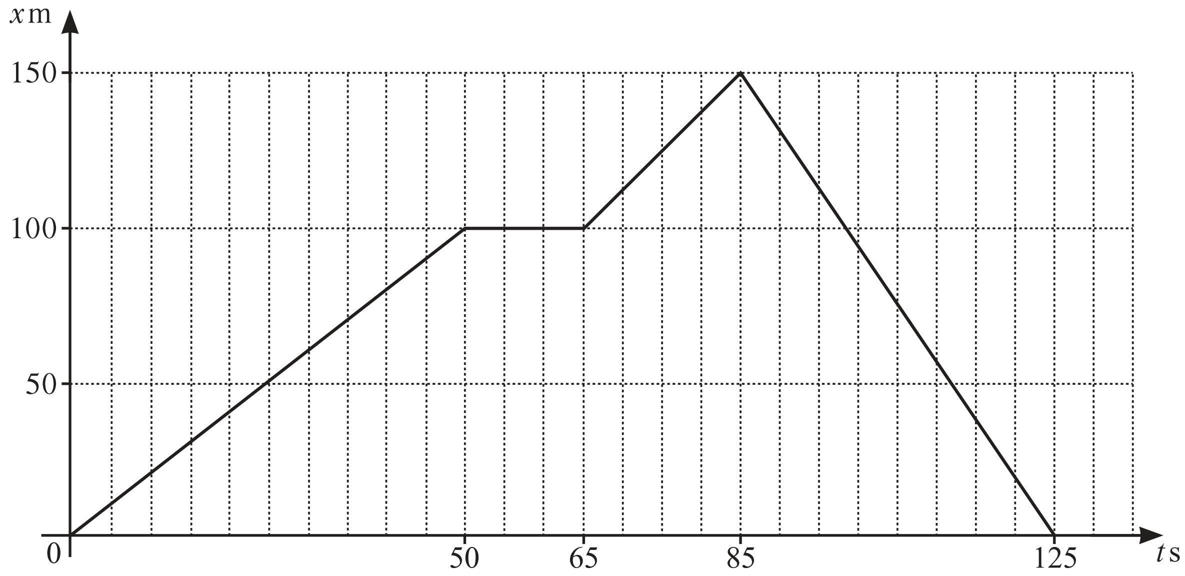
(ii) Find $f^{-1}(x)$ and its domain. [3]

(b) $g(x) = x + 5 \quad \text{for } x \in \mathbb{R}$

$h(x) = \sqrt{2x - 3} \quad \text{for } x \geqslant \frac{3}{2}$

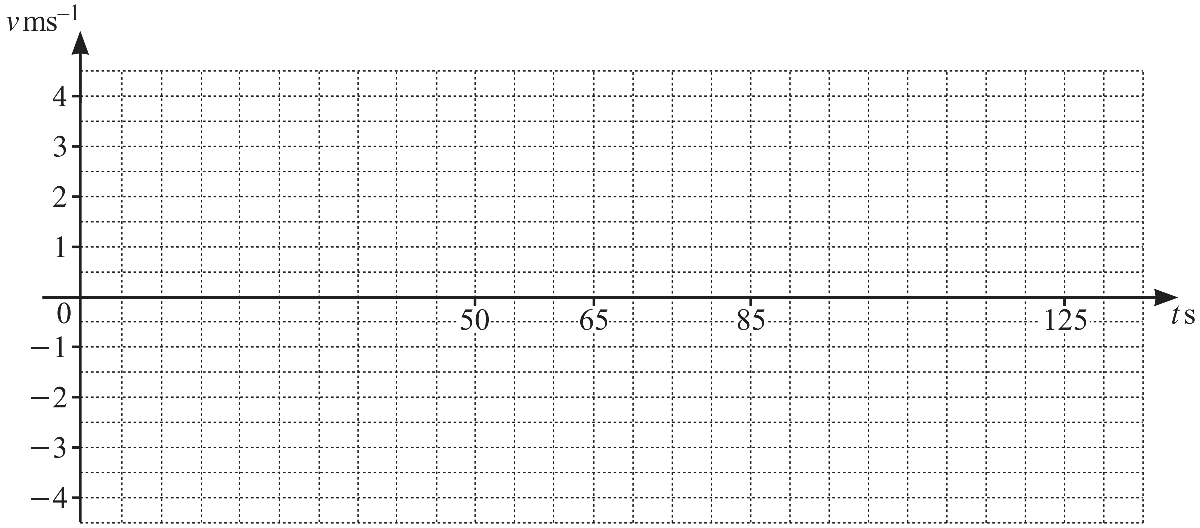
Solve $gh(x) = 7$. [3]

4 (a)



The diagram shows the x–t graph for a runner, where displacement, x, is measured in metres and time, t, is measured in seconds.

(i) On the axes below, draw the v–t graph for the runner. [3]



(ii) Find the total distance covered by the runner in 125 s. [1]

(b) The displacement, x m, of a particle from a fixed point at time t s is given by $x = 6\cos\left(3t + \frac{\pi}{3}\right)$. Find the acceleration of the particle when $t = \frac{2\pi}{3}$. [3]

5 Given that the coefficient of x^2 in the expansion of $(1+x)\left(1-\frac{x}{2}\right)^n$ is $\frac{25}{4}$, find the value of the positive integer n. [5]

6 It is known that $y = A \times 10^{bx^2}$, where A and b are constants. When $\lg y$ is plotted against x^2, a straight line passing through the points (3.63, 5.25) and (4.83, 6.88) is obtained.

(a) Find the value of A and of b. [4]

Using your values of A and b, find

(b) the value of y when $x = 2$, [2]

(c) the positive value of x when $y = 4$. [2]

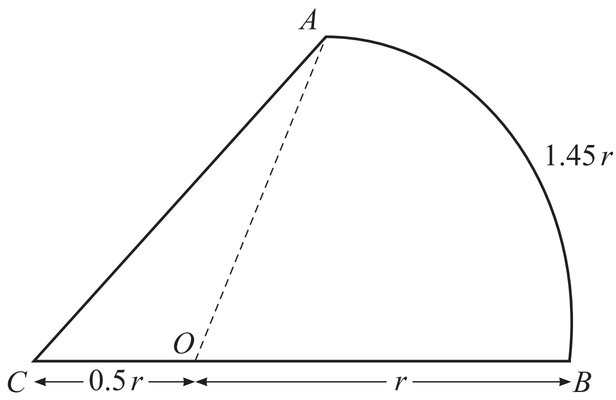
7 The polynomial $p(x) = ax^3 + bx^2 - 19x + 4$, where a and b are constants, has a factor $x + 4$ and is such that $2p(1) = 5p(0)$.

(a) Show that $p(x) = (x+4)(Ax^2 + Bx + C)$, where A, B and C are integers to be found. [6]

(b) Hence factorise $p(x)$. [1]

(c) Find the remainder when $p'(x)$ is divided by x. [1]

8 In this question all lengths are in centimetres.



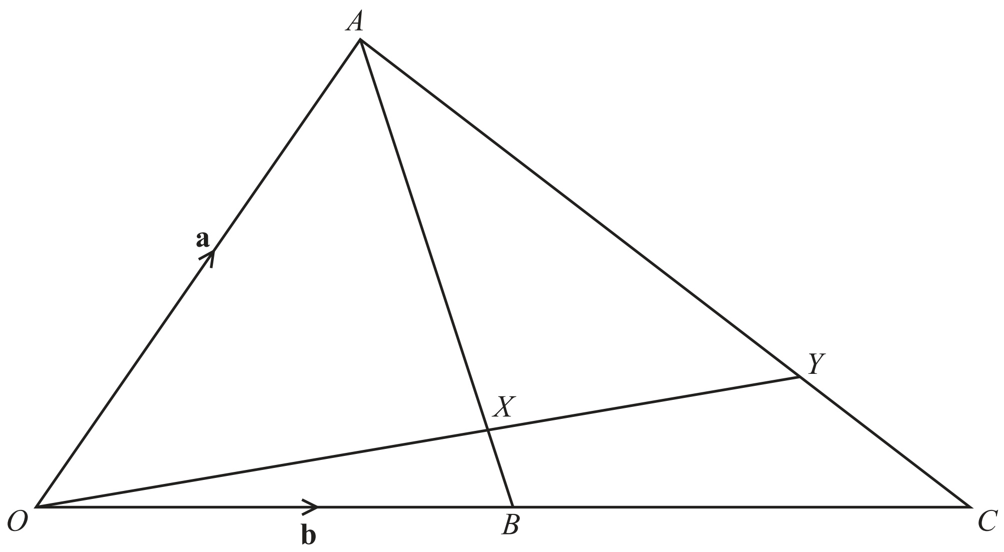
The diagram shows the figure ABC. The arc AB is part of a circle, centre O, radius r, and is of length $1.45r$. The point O lies on the straight line CB such that $CO = 0.5r$.

(a) Find, in radians, the angle AOB. [1]

(b) Find the area of ABC, giving your answer in the form kr^2, where k is a constant. [3]

(c) Given that the perimeter of ABC is 12 cm, find the value of r. [4]

9



The diagram shows the triangle OAC. The point B is the midpoint of OC. The point Y lies on AC such that OY intersects AB at the point X where $AX : XB = 3:1$. It is given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Find $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, giving your answer in its simplest form. [3]

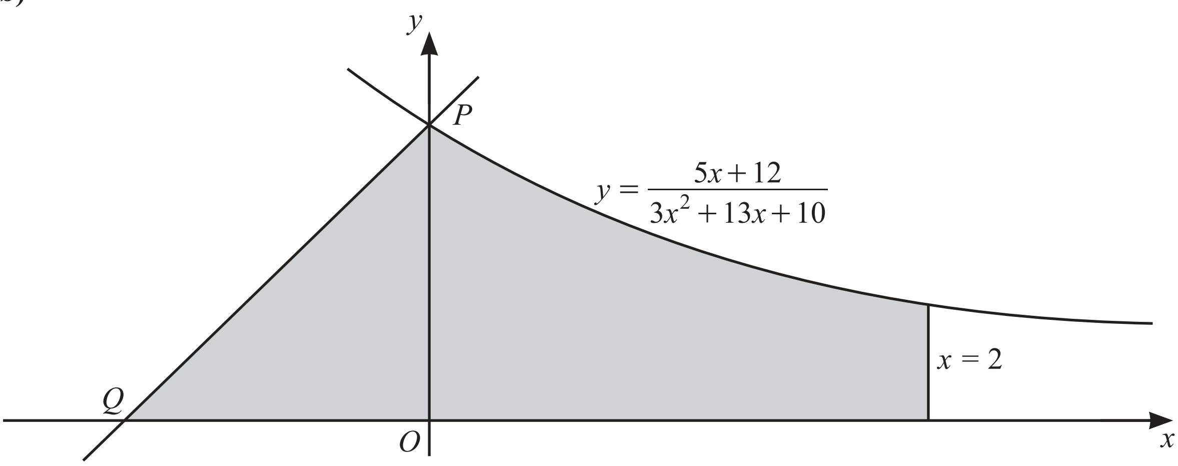
(b) Find $\overrightarrow{AC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [1]

(c) Given that $\overrightarrow{OY} = h\overrightarrow{OX}$, find $\overrightarrow{AY}$ in terms of **a**, **b** and h. [1]

(d) Given that $\overrightarrow{AY} = m\overrightarrow{AC}$, find the value of h and of m. [4]

10 **(a)** Show that $\frac{1}{x+1}+\frac{2}{3x+10}$ can be written as $\frac{5x+12}{3x^2+13x+10}$. [1]

(b)



The diagram shows part of the curve $y=\frac{5x+12}{3x^2+13x+10}$, the line $x=2$ and a straight line of gradient 1. The curve intersects the y-axis at the point P. The line of gradient 1 passes through P and intersects the x-axis at the point Q. Find the area of the shaded region, giving your answer in the form $a+\frac{2}{3}\ln\left(b\sqrt{3}\right)$, where a and b are constants. [9]

Additional working space for question 10

Question 11 is printed on the next page.

11 **(a)** Given that $2\cos x = 3\tan x$, show that $2\sin^2 x + 3\sin x - 2 = 0$. [3]

(b) Hence solve $2\cos\left(2\alpha + \frac{\pi}{4}\right) = 3\tan\left(2\alpha + \frac{\pi}{4}\right)$ for $0 < \alpha < \pi$ radians, giving your answers in terms of π. [4]